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From: A. Suter

E-Mail: andreas.suter@psi.ch

Homogenous Disorder Model: GbG in Longitudinal Fields

Noakes and Kalvius [1] derived a phenomenological model for homogenous disorder: Gaussian-broadened Gaussian disorder (see also Ref. [2]). In both mentioned references only the zero field case and the weak transverse field case are discussed. Here I briefly summarize the longitudinal field (LF) case under the assumption that the applied field doesn't polarize the impurities, *i.e.* the applied field is "innocent".

The Gauss-Kubo-Toyabe LF polarization function is

$$P_{Z,GKT}^{LF} = 1 - 2 \frac{\sigma^2}{\omega_{\text{ext}}^2} [1 - \cos(\omega_{\text{ext}} t) \exp(-1/2(\sigma t)^2)] + \quad (1)$$

$$+ 2 \frac{\sigma^2}{\omega_{\text{ext}}^3} \int_0^t \sin(\omega_{\text{ext}} \tau) \exp(-1/2(\omega_{\text{ext}} \tau)^2) d\tau. \quad (2)$$

The Gaussian disorder is assumed to have the functional form

$$\varrho = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_1} \exp\left(-\frac{1}{2} \left[\frac{\sigma - \sigma_0}{\sigma_1}\right]^2\right). \quad (3)$$

In Ref.[2] a slightly different notation is used: $\sigma \rightarrow \Delta_G$, $\sigma_0 \rightarrow \Delta_0$, and $\sigma_1 \rightarrow \Delta_{\text{GbG}}$. The GbG LF polarization function is given by

$$P_{Z,\text{GbG}}^{LF} = \int_0^\infty d\sigma \{ \varrho \cdot P_{Z,GKT}^{LF} \}. \quad (4)$$

Assuming that $\sigma_0 \gg \sigma_1$ this can be approximated by

$$P_{Z,\text{GbG}}^{LF} \simeq \int_{-\infty}^\infty d\sigma \{ \varrho \cdot P_{Z,GKT}^{LF} \}. \quad (5)$$

Integrating

$$P_{Z,\text{GbG}}^{LF,(1)} = \int_{-\infty}^\infty d\sigma \{ \varrho \cdot P_{Z,GKT}^{LF,(1)} \},$$

where $P_{Z,GKT}^{LF,(1)}$ is given by Eq.(1), leads to

$$P_{Z,\text{GbG}}^{LF,(1)} = 1 - 2 \frac{\sigma_0^2 + \sigma_1^2}{\omega_{\text{ext}}^2} + 2 \frac{\sigma_0^2 + \sigma_1^2(1 + \sigma_1^2 t^2)}{\omega_{\text{ext}}^2(1 + \sigma_1^2 t^2)^{5/2}} \cos(\omega_{\text{ext}} t) \exp\left[-\frac{1}{2} \frac{\sigma_0^2 t^2}{1 + \sigma_1^2 t^2}\right], \quad (6)$$

and Eq.(2) leads to the non-analytic integral

$$\begin{aligned} P_{Z,\text{GbG}}^{LF,(2)} &= \int_{-\infty}^\infty d\sigma \{ \varrho \cdot P_{Z,GKT}^{LF,(2)} \} \\ &= \int_0^t d\tau \left\{ \frac{\sigma_0^4 + 3\sigma_1^4(1 + \sigma_1^2 \tau^2)^2 + 6\sigma_0^2 \sigma_1^2(1 + \sigma_1^2 \tau^2)}{\omega_{\text{ext}}^3(1 + \sigma_1^2 \tau^2)^{9/2}} \sin(\omega_{\text{ext}} \tau) \exp\left[-\frac{1}{2} \frac{\sigma_0^2 \tau^2}{1 + \sigma_1^2 \tau^2}\right] \right\}. \end{aligned} \quad (7)$$

The full GbG LF polarization function is hence

$$P_{Z,\text{GbG}}^{\text{LF}} = P_{Z,\text{GbG}}^{\text{LF},(1)} + P_{Z,\text{GbG}}^{\text{LF},(2)} \quad (8)$$

The GbG LF Polarization Function as a User Function in MUSRFIT

Eqs.(6)&(7) are implemented in MUSRFIT as user function. The current implementation is far from being efficient but stable. The typical call from within the msr-file would be

```
#####
FITPARAMETER
#      Nr. Name      Value      Step      Pos_Error  Boundaries
      1 PlusOne      1          0          none
      2 MinusOne     -1          0          none
      3 Alpha        0.78699    -0.00036   0.00036    0          none
      4 Asy          0.06682    0.00027   none       0          0.33
      5 Sig0         0.3046     -0.0087   0.0093     0          100
      6 Rb           1.0000     0.0027   none       0          1
      7 Field0       0          0          none
      8 Field1       20.03      0          none
      9 Field2       99.32      0          none

#####
THEORY
asymmetry fun1
userFcn libGbGLF PGbGLF map2 5 fun2 (field sigma0 Rb)

#####
FUNCTIONS
fun1 = map1 * par4
fun2 = par5 * par6
```

where PGbGLF takes 3 arguments:

1. field in Gauss
2. σ_0 in $(1/\mu s)$
3. $R_b = \sigma_1/\sigma_0$

Be aware that we explicitly assumed $\sigma_1 \ll \sigma_0$, *i.e.* $R_b < 1$.

References

- [1] D. R. Noakes, G. M. Kalvius, Phys. Rev. B, **56**, 2352 (1997).
- [2] A. Yaouanc, P. Dalmas de Réotier, “Muon Spin Rotation, Relaxation, and Resonance”, Oxford University Press (2011).