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Memorandum

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Homogenous Disorder Model: GbG in Longitudinal Fields

Noakes and Kalvius [1] derived a phenomenological model for homogenous disorder: Gaussian-broadened Gaussian disorder (see also Ref. [2]). In both mentioned references only the zero field case and the weak transverse field case are discussed. Here I briefly summarize the longitudinal field (LF) case under the assumption that the applied field doesn't polarize the impurties, *i.e.* the applied field is "innocent".

The Gauss-Kubo-Toyabe LF polarization function is

$$P_{Z,\mathrm{GKT}}^{\mathrm{LF}} = 1 - 2\frac{\sigma^2}{\omega_{\mathrm{ext}}^2}\left[1-\cos(\omega_{\mathrm{ext}}t)\,\exp\left(-1/2(\sigma t)^2\right)\right] + \tag{1}$$

$$+2\frac{\sigma^2}{\omega_{\mathrm{ext}}^3}\int_0^t \sin(\omega_{\mathrm{ext}}\tau)\,\exp\left(-1/2(\omega_{\mathrm{ext}}\tau)^2\right)d\tau. \tag{2}$$

The Gaussian disorder is assumed to have the funtional form

$$\varrho = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_1}\exp\left(-\frac{1}{2}\left[\frac{\sigma-\sigma_0}{\sigma_1}\right]^2\right). \tag{3}$$

In Ref.[2] a slightly different notation is used: $\sigma \to \Delta_{\mathrm{G}}$, $\sigma_0 \to \Delta_0$, and $\sigma_1 \to \Delta_{\mathrm{GbG}}$.
The GbG LF polarization function is given by

$$P_{Z,\mathrm{GbG}}^{\mathrm{LF}} = \int_0^\infty d\sigma\left\{\varrho\cdot P_{Z,\mathrm{GKT}}^{\mathrm{LF}}\right\}. \tag{4}$$

Assuming that $\sigma_0 \gg \sigma_1$ this can be approximated by

$$P_{Z,\mathrm{GbG}}^{\mathrm{LF}} \simeq \int_{-\infty}^\infty d\sigma\left\{\varrho\cdot P_{Z,\mathrm{GKT}}^{\mathrm{LF}}\right\}. \tag{5}$$

Integrating

$$P_{Z,\mathrm{GbG}}^{\mathrm{LF},(1)} = \int_{-\infty}^\infty d\sigma\left\{\varrho\cdot P_{Z,\mathrm{GKT}}^{\mathrm{LF},(1)}\right\},$$

where $P_{Z,\mathrm{GKT}}^{\mathrm{LF},(1)}$ is given by Eq.(1), leads to

$$P_{Z,\mathrm{GbG}}^{\mathrm{LF},(1)} = 1 - 2\frac{\sigma_0^2+\sigma_1^2}{\omega_{\mathrm{ext}}^2} + 2\frac{\sigma_0^2+\sigma_1^2(1+\sigma_1^2t^2)}{\omega_{\mathrm{ext}}^2(1+\sigma_1^2t^2)^{5/2}}\cos(\omega_{\mathrm{ext}}t)\,\exp\left[-\frac{1}{2}\frac{\sigma_0^2t^2}{1+\sigma_1^2t^2}\right], \tag{6}$$

and Eq.(2) leads to the non-analytic integral

$$\begin{aligned}P_{Z,\mathrm{GbG}}^{\mathrm{LF},(2)} &= \int_{-\infty}^\infty d\sigma\left\{\varrho\cdot P_{Z,\mathrm{GKT}}^{\mathrm{LF},(2)}\right\}\\ &= \int_0^t d\tau\left\{\frac{\sigma_0^4+3\sigma_1^4(1+\sigma_1^2\tau^2)^2+6\sigma_0^2\sigma_1^2(1+\sigma_1^2\tau^2)}{\omega_{\mathrm{ext}}^3(1+\sigma_1^2\tau^2)^{9/2}}\sin(\omega_{\mathrm{ext}}\tau)\,\exp\left[-\frac{1}{2}\frac{\sigma_0^2t^2}{1+\sigma_1^2t^2}\right]\right\}.\end{aligned} \tag{7}$$

The full GbG LF polarization function is hence

$$P_{Z,\mathrm{GbG}}^{\mathrm{LF}} = P_{Z,\mathrm{GbG}}^{\mathrm{LF},(1)} + P_{Z,\mathrm{GbG}}^{\mathrm{LF},(2)} \tag{8}$$

The GbG LF Polarization Function as a User Function in MUSRFIT

Eqs.(6)&(7) are implemented in MUSRFIT as user function. The current implementation is far from being efficient but stable. The typical call from within the msr-file would be

```
###############################################################
FITPARAMETER
#      Nr. Name        Value     Step      Pos_Error  Boundaries
        1 PlusOne     1         0          none
        2 MinusOne    -1        0          none
        3 Alpha       0.78699   -0.00036   0.00036     0       none
        4 Asy         0.06682   0.00027    none        0       0.33
        5 Sig0        0.3046    -0.0087    0.0093      0       100
        6 Rb          1.0000    0.0027     none        0       1
        7 Field0      0         0          none
        8 Field1      20.03     0          none
        9 Field2      99.32     0          none

###############################################################
THEORY
asymmetry   fun1
userFcn  libGbGLF PGbGLF map2 5 fun2   (field sigma0 Rb)

###############################################################
FUNCTIONS
fun1 = map1 * par4
fun2 = par5 * par6
```

where `PGbGLF` takes 3 arguments:

1. field in Gauss
2. σ_0 in $(1/\mu s)$
3. $R_b = \sigma_1/\sigma_0$

Be aware that we explicitly assumed $\sigma_1 \ll \sigma_0$, *i.e.* $R_b < 1$.

References

[1] D. R. Noakes, G. M. Kalvius, Phys. Rev. B, **56**, 2352 (1997).

[2] A. Yaouanc, P. Dalmas de Réotier, "Muon Spin Rotation, Relaxation, and Resonance", Oxford University Press (2011).

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